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Impedance Transformation using a $\lambda/4$ Line

Design Criteria using a Smith Diagram or BASIC Program

A line or a waveguide will "transform" a real or reactive impedance or a combination of the two. According to the impedance of the transformation line, it is possible to transform a low real impedance to a high impedance, or a high impedance to a low impedance. An inductive reactive component will be converted to a capacitive, or a capacitive to an inductive component.

The latter is especially of interest in waveguide technology; in this case, screws can be inserted into the waveguide at a certain distance from a point of discontinuity, for instance at a spacing of $\lambda/4$. The waveguide transforms the capacitive effect of the screw (see **Figure 1**) to an inductive component at the position of discontinuity.

The real impedance transformation using a $\lambda/4$ line is well known: The required impedance is obtained from the simple equation:

$$Z = \sqrt{Z_{\text{out}} \times Z_{\text{in}}}$$

where Z_{out} is the terminating, and Z_{in} is the required input impedance of the $\lambda/4$ line.

Example: Four parallel-connected 50Ω resistors are to be matched to the output of a transmitter having an output impedance of 50Ω (for instance four antennas at 144 MHz). Required is the Z of the transformation line.

Solution: The four consumers form a terminating resistance of $Z_{\text{out}} = 12.5 \Omega$. The Z_{in} of the $\lambda/4$ transformer should coincide to the output impe-

dance Z_{out} of the transmitter: 50Ω . The following will result:

$$Z = \sqrt{12.5 \times 50 \Omega} = 25 \Omega$$

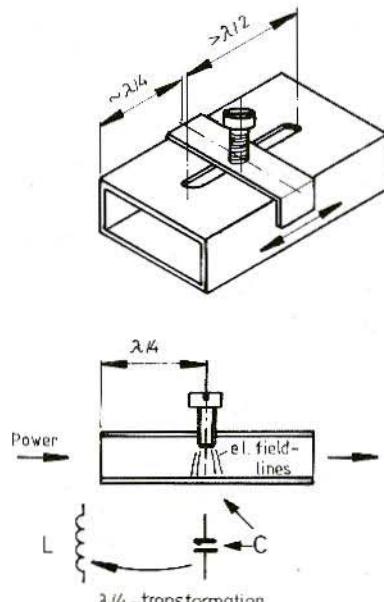


Fig. 1: Operation of the sliding screw tuner

This $\lambda/4$ -transformer can be obtained by connecting two $50\ \Omega$ cables in parallel. Assuming a velocity factor of 0.66, this will result in the following for the 144 MHz band: $0.5\ \text{m} \times 0.66 = 0.33\ \text{m}$.

If Z and Z_{out} are given, Z_{in} will result by conversion of the equation:

$$Z_{\text{in}} = Z^2/Z_{\text{out}}$$

An input reactive impedance can be calculated in a similar manner if the $\lambda/4$ transformer is terminated with a pure, reactive impedance X_{out} :

$$X_{\text{in}} = Z^2/X_{\text{out}}$$

The equation does not give any information regarding the sign (plus in the case of an inductive, and minus in the case of a capacitive reactive component). One will have to remember that a capacitive reactive component X_{out} will result in an inductive reactive component X_{in} , and vice versa (Figure 1). The sign would be available only when doing a complex calculation!

Example: A 12 cm long line ($\lambda/4$ at 433 MHz assuming a velocity factor of approx. 0.66) of $Z = 50\ \Omega$ is terminated with $C = 4.7\ \text{pF}$. Which L will result at 433 MHz? (with $4.7\ \text{pF}$, X_{out} is approx. $-78\ \Omega$).

Solution: $X_{\text{in}} = (50^2/78)\ \Omega$. The sign “-” of the capacitive reactive component of X_{out} is not used. X_{in} is inductive and corresponds to an $L = 11.7\ \text{nH}$ at 433 MHz.

The given equations are no longer suitable when the line is terminated with a series circuit of real and reactive impedances. A clear method can be found when using a Smith diagram, if one is used to working with it: The terminating impedance comprising a real and reactive impedance is standardized to the impedance Z of the $\lambda/4$ -line to be used and inserted (W_{out}). The resulting input impedance is found in the Smith diagram diametrically opposite (W_{in}).

Example: The input connector of a power amplifier is connected via a $\lambda/4$ stripline with an impedance of $20\ \Omega$ to the relatively low input impedance of the power transistor of, for instance, $5\ \Omega$ in series with an (inductive) reactive impedance of $+4\ \Omega$. The two impedances form thus Z_{out} and X_{out} of the stripline and are transformed to a certain impedance at the input connector. The transformed values Z_{in} and X_{in} are to be calculated!

Solution: Standardize Z_{out} and Z_{in} (in other words divide by the Z of the line):

$$Z_{\text{out}}' = 5\ \Omega/20\ \Omega = 0.25; X_{\text{out}}' = +4\ \Omega/20\ \Omega = +0.2.$$

Both values are now inserted into the circular coordinates of the Smith diagram (Figure 2); (a positive X_{out}' means “inductive” which means the upper part of the diagram).

The result is point W_{out}' . Diametrically one will obtain W_{in}' . Attention should be paid that the straight line really passes through the center of the horizontal line. This means that W_{in}' will have the same spacing from the center point “1” as W_{out}' . One then reads off the coordinates of W_{in} as:

$$Z_{\text{in}}' = 2.5 \text{ and } X_{\text{in}}' = -2.$$

The negative sign represents a capacitive reactive component of W_{in}' . The values are “destandardized” by multiplication with the standard impedance, and one will obtain the input impedance as:

$$Z_{\text{in}} = 2.5 \times 20\ \Omega = 50\ \Omega, X_{\text{in}} = -2 \times 20\ \Omega = -40\ \Omega \\ (Z_{\text{in}} \text{ with } X_{\text{in}} \text{ in series!})$$

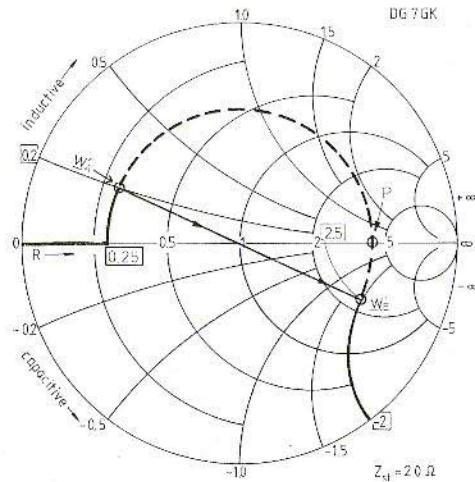


Fig. 2: Impedance transformation of W_{out} to W_{in} using a $\lambda/4$ line

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5 PRINT"U"
10 PRINT;PRINT;PRINT
20 PRINT"EINGANGSIMPED. DES LAMBDA-VIERTEL-TRAFOS":PRINT
30 PRINT" BY G. STADLER":FOR I=1 TO 2000:NEXT I:PRINT"U"
40 INPUT"GEBEN SIE WELLENWIDERSTAND EIN":ZI:PRINT;PRINT;PRINT
50 INPUT"GEBEN SIE ABSCHLUSS-WIRKWIDERSTAND EIN":RA:PRINT;PRINT;PRINT
60 INPUT"GEBEN SIE ABSCHLUSS-BLINDWIDERSTAND EIN":XA:PRINT"U"
70 IF RA=0 AND XA=0 THEN PRINT" RE=UNENDLICH":GOTO 150
80 ZE=ZA2/(RA^2+XA^2)^(1/2)
90 IF RA>0 THEN GOTO 110
100 IF RA=0 THEN A=SGN(XA)*(PI/2):GOTO 130
110 K=XA/RA
120 A=ATN(K)
130 PRINT" RE=ZE*COS(A):PRINT;PRINT;PRINT
140 PRINT" XE=(-1)*ZE*SIN(A)
150 END
READY.

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Fig. 3: BASIC program for calculation of the components of the input impedance of a $\lambda/4$ line as a function of the terminating impedance (impedance values in Ω , with input and output in numerical values)

NOTE: with respect to the previous example:
 In certain circumstances, it may be advisable to make a semicircle in a clockwise direction from W_{out} (as shown in Figure 2 as a dashed line) around "1", instead of plotting a straight line: In this manner, it is also possible to determine point W_{in} . The semicircle forms the "transformation path" of the line! This means that if the line is shorter than $\lambda/4$, the transformation path will also be shorter than the semicircular arc. In this case, there is a proportional relationship between the length of the arc and line length. It is possible by selection of the line length to ensure that the transformation can, for instance, end on the horizontal axis of the Smith diagram. In the case of Figure 2, this corresponds to point P. In this case, the result will be that the input impedance is a pure real component: Point P is approximately 4.2, which when destandardized results in a Z_{in} of $4.2 \times 20 \Omega = 84 \Omega$. This would allow the power transistor to be well matched to a system with an impedance of 50Ω ($VSWR = 84 \Omega / 50 \Omega = 1.7$). It is, of course, possible to improve the matching still further by reducing the impedance of the stripline

(recommendation: repeat the above example with an impedance value of $Z = 15 \Omega$!).

Concentric circles in the Smith diagram replace a relatively complex calculation of the input impedance of a line. A BASIC program is shown in **Figure 3** in which the equation is programmed for the special case of a $\lambda/4$ line. The real and reactive impedances are inserted and result as numerical values in Ω . Attention should be paid that one lists capacitive reactive impedances with a minus sign. The program is, however, only suitable for series circuits. The same is valid, of course, when using the Smith diagram.

Since the accuracy of the computer is very often not required – possibly due to the tolerances of the impedances used – it may be of advantage to use the Smith diagram. This is also valid for applications in waveguide technology. It is then possible for reflection factors to be inserted into the diagram according to amount and angle. A standardization or destandardization is not required since the wave of a waveguide cannot be defined as of a specific impedance.

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